

Mass Determination Method for $\tilde{e}_{L,R}$ Above Production Threshold

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 (February 7, 2008)

The determination of the masses of Supersymmetric particles such as the selectron, for energies above threshold using the energy end-points method is subject to signal deconvolution difficulties and to Standard Model and Supersymmetry backgrounds. The important features of $\tilde{e}_{R,L}^\pm$ production are used to design an experimentally robust method, with good resolution, both for determining the $\tilde{e}_{R,L}$ and the $\tilde{\chi}_1^0$ masses and for suppressing backgrounds. Additional features, such as the determination of the relative leptonic branching ratios of the selectron are present in the method.

Keywords: SUSY Particle Production, Experimental Mass Determination, Selectron

The determination of Supersymmetric particle masses using the energy end-point method [1] is well known. The measurement of selectron masses is subject to two experimental difficulties: on the one hand the energy distribution of the visible particles in the event is an overlap of 4 “box like” distributions due to the production channels $\tilde{e}_R^+\tilde{e}_R^-$, $\tilde{e}_L^+\tilde{e}_L^-$, $\tilde{e}_R^+\tilde{e}_L^-$, $\tilde{e}_L^+\tilde{e}_R^-$ and on the other, the Supersymmetry (SUSY) signal is masked by very large Standard Model (SM) backgrounds, such as W^+W^- and $\gamma^*\gamma^*$. The consequences are, for the former the difficulty of resolving overlapping “box”-edges in the lower energy region making it hard to determine which edge is due to which SUSY particle decay, while for the latter the masking of the SUSY-signal by large SM backgrounds.

Many other studies [2–6] of the determination of supersymmetric masses via the energy spectrum of the observed particles have been carried out showing the usefulness of the technique and the levels of accuracy possible. The complications in the selectron spectrum are being solved here.

During Snowmass-2001 we realised that the difference of the observed positron and electron (from the $\tilde{e}_{L,R}$ decay) energy distributions enhanced by the difference in cross sections with incident electron polarization (possible only with a Linear Collider) can solve these problems and provide a method by which we can determine the masses unambiguously with good resolution. In addition it offers new features, built in redundancy, the determination of the partial leptonic branching ratios to the channels $\tilde{\chi}_1^\pm\nu_e$ and $\tilde{\chi}_1^0e$. Information about the $\tilde{\chi}_1^\pm$ and $\tilde{\tau}_1^\pm$ masses may, in principle, also be determined.

The e^+, e^- energy distributions difference is given by:

$$\Delta(E) = Lumi \cdot (\sigma_{RL} - \sigma_{LR}) \cdot [R'_{box}(E) - L'_{box}(E)] \quad (1)$$

where, for instance, σ_{LR} is the production cross section for $\tilde{e}_L^+\tilde{e}_R^-$, while $R'(E)$ and $L'(E)$ are the appropriate energy box distributions (for the incident electron right-handed and left-handed polarization) each normalised to

unity. The method presented here removes the contribution to the energy spectrum from the reactions producing $\tilde{e}_R^+\tilde{e}_R^-$ and $\tilde{e}_L^+\tilde{e}_L^-$, which solves the mass measurement difficulty and reduces the mass errors.

Asymmetric boosts are present when \tilde{e}_L^\pm and \tilde{e}_R^\pm are produced in the same reaction. The values of the boosts being in this case:

$$\begin{aligned} \gamma_{\tilde{e}_L} &= \frac{1}{2M_{\tilde{e}_L}}\sqrt{s} + \frac{M_{\tilde{e}_L}^2 - M_{\tilde{e}_R}^2}{2M_{\tilde{e}_L}\sqrt{s}} \\ \gamma_{\tilde{e}_R} &= \frac{1}{2M_{\tilde{e}_R}}\sqrt{s} + \frac{M_{\tilde{e}_R}^2 - M_{\tilde{e}_L}^2}{2M_{\tilde{e}_R}\sqrt{s}} \end{aligned} \quad (2)$$

The energies of the electron and positron which are the particles visible in the event are related to the decay CM energy. Since $m_e \ll M_{L,R}$ we obtain the lower and higher bounds of the electron/positron energy distribution in the LAB reference frame:

$$\begin{aligned} E_{e^\pm}(HI) &= E_{e^\pm}^*(\gamma + \sqrt{\gamma^2 - 1}) \\ E_{e^\pm}(LO) &= E_{e^\pm}^*(\gamma - \sqrt{\gamma^2 - 1}) = E_{e^\pm}^*/(\gamma + \sqrt{\gamma^2 - 1}) \end{aligned} \quad (3)$$

Solving for $E_{e^\pm}^*$ and γ we obtain:

$$\begin{aligned} E_{e^\pm}^* &= \sqrt{E_{e^\pm}(HI) \cdot E_{e^\pm}(LO)} \\ \lambda_\pm &= \sqrt{\frac{E_{e^\pm}(HI)}{E_{e^\pm}(LO)}} \end{aligned} \quad (4)$$

where $\lambda = \gamma + \sqrt{\gamma^2 - 1}$, and $\gamma = \lambda/2 + 1/(2\lambda)$. We can determine the relationship between the mass errors and the energy-edge errors. To a very good approximation this follows from:

$$\frac{\delta M_{\tilde{e}_{L,R}}}{M_{\tilde{e}_{L,R}}} = - \frac{\delta \gamma_{\tilde{e}_{L,R}}}{\gamma_{\tilde{e}_{L,R}}} = \frac{\lambda_{\tilde{e}_{L,R}}^2 - 1}{\lambda_{\tilde{e}_{L,R}}^2 + 1} \left(\frac{\delta \lambda_{\tilde{e}_{L,R}}}{\lambda_{\tilde{e}_{L,R}}} \right) \quad (5)$$

This becomes to a good approximation

$$\frac{\delta M}{M} = \frac{1}{\sqrt{2}} \frac{E_e(HI) - E_e(LO)}{E_e(HI) + E_e(LO)} \frac{\delta(E_e)}{E_e} \quad (6)$$

where we assume that $\delta(E_e)/E_e$ is the same at the low and high end of the energy spectrum.

The complete set of equations for determining the masses from the energy spectrum end-points is:

$$\begin{aligned} 2\gamma_{\tilde{e}_R} M_{\tilde{e}_R} \sqrt{s} &= \sqrt{s}^2 + (M_{\tilde{e}_R}^2 - M_{\tilde{e}_L}^2) \\ 2\gamma_{\tilde{e}_L} M_{\tilde{e}_L} \sqrt{s} &= \sqrt{s}^2 + (M_{\tilde{e}_L}^2 - M_{\tilde{e}_R}^2) \\ 2E_{e,R}^* M_{\tilde{e}_R} &= M_{\tilde{e}_R}^2 - M_{\tilde{\chi}_1^0}^2 \\ 2E_{e,L}^* M_{\tilde{e}_L} &= M_{\tilde{e}_L}^2 - M_{\tilde{\chi}_1^0}^2 \end{aligned} \quad (7)$$

The system, although quadratic, allows one to substitute the quadratic terms and reach a linear solution:

$$\begin{aligned} M_R &= \frac{1}{2} \sqrt{s} \frac{\gamma_{\tilde{e}_L} \sqrt{s} + 2E_{e,L}^*}{\gamma_{\tilde{e}_L} \gamma_{\tilde{e}_R} \sqrt{s} + \gamma_{\tilde{e}_R} E_{e,L}^* + \gamma_{\tilde{e}_L} E_{e,R}^*} \\ M_L &= \frac{1}{2} \sqrt{s} \frac{\gamma_{\tilde{e}_R} \sqrt{s} + 2E_{e,R}^*}{\gamma_{\tilde{e}_L} \gamma_{\tilde{e}_R} \sqrt{s} + \gamma_{\tilde{e}_L} E_{e,R}^* + \gamma_{\tilde{e}_R} E_{e,L}^*} \end{aligned} \quad (8)$$

The fit process that searches for the masses $M_{\tilde{e}_L}$, $M_{\tilde{e}_R}$ and $M_{\tilde{\chi}_1^0}$ is carried out with MINUIT [7] that obtains the best solution to the set of equations above.

The properties of the above defined distribution are quite simple and robust. First, there are only 2 (well separated) “boxes” as seen in Fig. 1; one positive, one negative, with an overlapping mid-region.

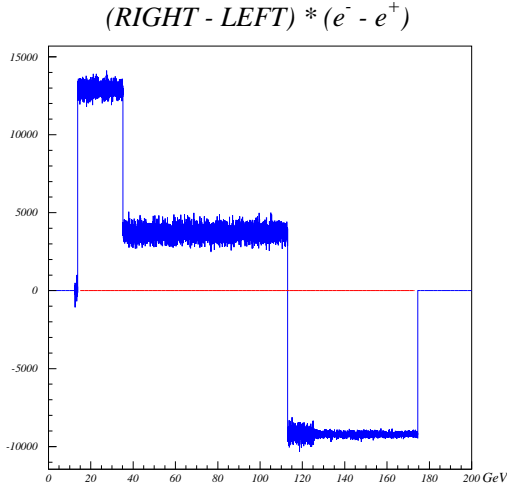


FIG. 1. Energy distributions difference between e^+ and e^- for only selectron decays. Using the difference of the difference, between *Right*_{80%} and *Left*_{80%} e^- beam polarisations, the differences are enhanced and any detector asymmetries between charged tracks are effectively cancelled.

This means that the edges can be comfortably resolved, as none of them overlap in any energy range. Secondly,

the edge-to-particle assignments are clear, the positive part of the distribution corresponding to \tilde{e}_R^\pm . Thirdly, since all SM backgrounds have the same energy distribution for the visible e^+ and e^- , their difference does not exhibit any contribution from Standard Model backgrounds. Even if there is a detector asymmetry between positive and negative tracks, this is eliminated through the use of the polarised version of the above difference as described in eqn. 1 and shown in Fig. 1.

It should be noted also that even though the backgrounds are reduced to the level of their statistical fluctuations, they can still be large and affect the signal. Hence removal of the background to first order by kinematical cuts or strategically located detector vetoes is still useful. It has been shown [8] that the $\gamma^* \gamma^*$ background can be suppressed to almost zero using these techniques. These cuts fail however when $\tilde{e}_{L,R}^\pm$ and $\tilde{\chi}_1^0$ are close in mass, for this case a Very-Forward Veto device is crucial. The method proposed here greatly alleviates the kinematic severity of the cuts needed to control this background.

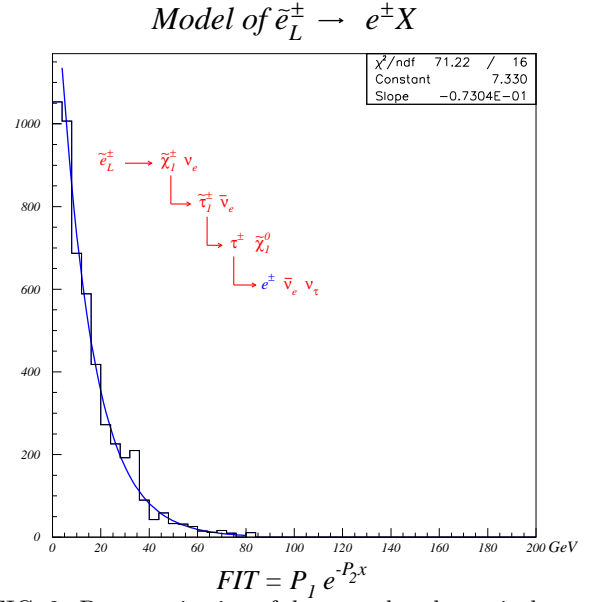


FIG. 2. Parametrisation of the secondary leptonic decays of the $\tilde{\chi}_1^\pm$. The chosen model was an exponential, which requires one shape parameter.

The procedure is complicated slightly by the fact that for a region of the SUSY parameter space \tilde{e}_L^\pm has two leptonic decay channels, one involving $\tilde{\chi}_1^\pm \nu_e$, and one $\tilde{\chi}_1^0 e^\pm$. It is assumed that the secondary $\tilde{\chi}_1^\pm$ hadronic decays can be eliminated, however the leptonic decays with $\tilde{\chi}_1^\pm \nu_e$ have the same signature, an e^\pm in the final state as the preferred $\tilde{\chi}_1^0 e^\pm$ decays and have to be accommodated in the fit. The difference distribution becomes in this case:

$$\Delta(E) = Lumi \cdot (\sigma_{RL} - \sigma_{LR}) \cdot (f + f').$$

$$\left[R'_{box}(E) - \frac{f}{f+f'} L'_{box}(E) - \frac{f'}{f+f'} X'_{box}(E) \right] \quad (9)$$

where $X'(E)$ is the energy distribution of the visible e^\pm from the secondary $\tilde{\chi}_1^\pm$ decay, and f and f' are the branching fractions to $L'(E)$ and $X'(E)$ respectively. The power of the method in its initial form is that this histogram has to be normalised to zero, and hence any pedestal can be determined and subtracted. It also has 2 normalisation conditions related to the $R'(E)$ and $L'(E)$ boxes, that connect the edge positions with the box-heights. To first order there are only 3 free parameters in the fit: $M_{\tilde{e}_L}$, $M_{\tilde{e}_R}$ and $M_{\tilde{\chi}_1^0}$. This provides a very robust fit. In the present case, the $X'(E)$ distribution (figure 2) has to be parametrised, and this adds an extra parameter to the fit. The chosen model was an exponential shape for $X'(E)$, hence only one extra shape parameter. The general idea of the simple fit remains: a robust fit (figure 3) with few, tightly bound parameters. The fit yields also the $f/(f+f')$ and $f'/(f+f')$ relative leptonic branching fractions for the $\tilde{\chi}_1^\pm \nu_e$ and $\tilde{\chi}_1^0 e$. The $X'(E)$ shape parameter gives general information about the $\tilde{\chi}_1^\pm$ and $\tilde{\tau}_1^\pm$ masses.

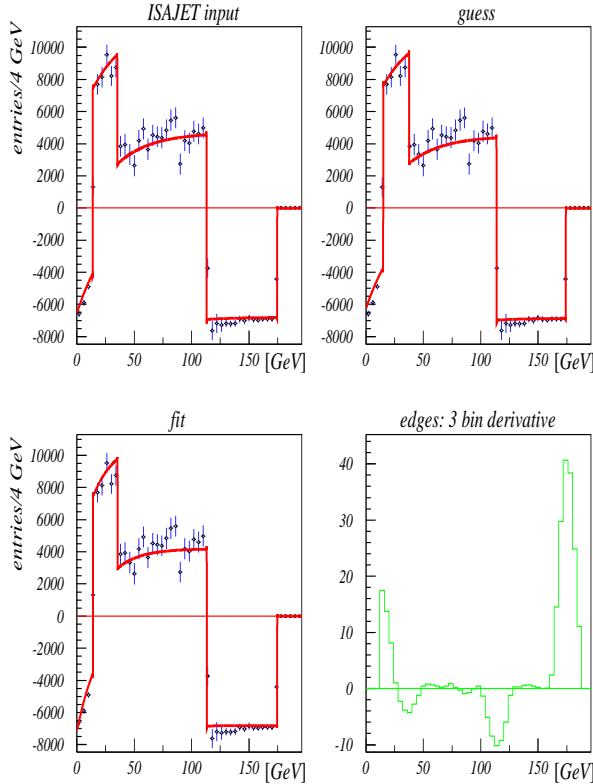


FIG. 3. Fit to the electron energy distribution including the $\tilde{\chi}_1^\pm$ decay mode of the \tilde{e}_L based on an integrated luminosity of 2000 fb^{-1} and a collision energy of 500 GeV. The input masses (in GeV) are $M_{\tilde{e}_R} = 142.96$, $M_{\tilde{e}_L} = 202.07$, $M_{\tilde{\chi}_1^0} = 95.70$. The fit gives $M_{\tilde{e}_R} = 143.12 \pm 0.23$, $M_{\tilde{e}_L} = 202.27 \pm 0.11$, $M_{\tilde{\chi}_1^0} = 95.72 \pm 0.12$.

This analysis is carried out for the MSSM parameters $m_0=100 \text{ GeV}$, $m_{1/2}=250 \text{ GeV}$, $A_0=0$, $\tan(\beta)=10$, $\mu=\text{positive}$. The resultant masses (in GeV) of the super-symmetric particles in our study is $m_{\tilde{e}_R}=143$, $m_{\tilde{e}_L}=202$, $m_{\tilde{\chi}_1^0}=96$, $m_{\tilde{\chi}_1^\pm}=174$, and $m_{\tilde{\tau}_1^\pm}=135$. The errors in the mass determination *versus* luminosity are shown in figure 4 for the “idealistic” case with no $\tilde{\chi}_1^\pm \nu_e$ channel, as well as for the “realistic” case including the $\tilde{\chi}_1^\pm \nu_e$. An indication of saturation of the mass-errors with luminosity is shown by the fit. The luminosity at which the error reaches 80% of its limiting value gives a range of 500 - 1000 fb^{-1} as the saturation limit. It can be seen that the largest difference in mass-errors between the “ideal” and “real” case is for $M_{\tilde{e}_R}$ due to its corresponding energy-“box” neighboring the perturbing $X'(E)$ distribution.

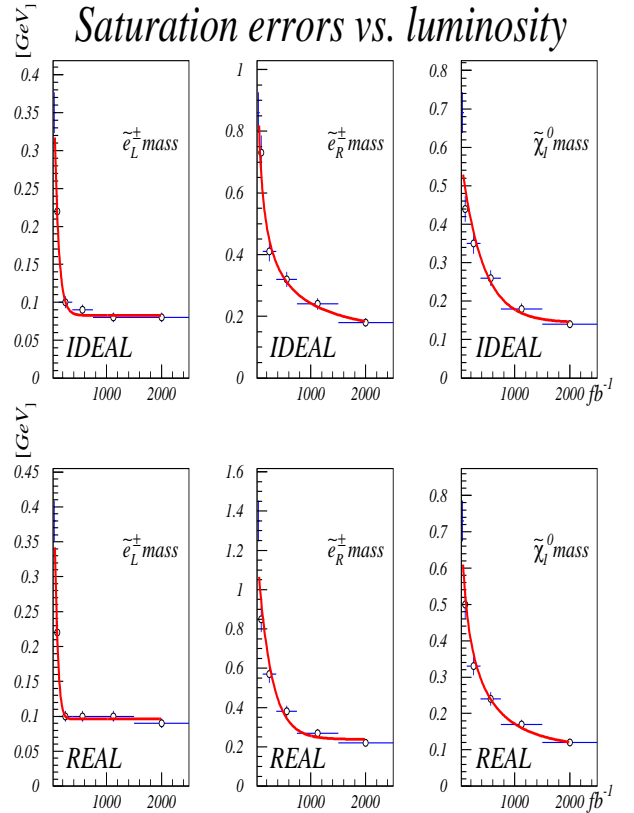


FIG. 4. Mass determination errors *versus* luminosity, for the prototype distribution that includes only the $\tilde{\chi}_1^0 e$ decay and for the realistic one including also the $\tilde{\chi}_1^\pm \nu_e$ decay. An indication of saturation with luminosity is seen.

The method is very robust and stable, its results being not only the masses involved, but also possibly the relative leptonic branching ratios, and some tangential information about the $\tilde{\chi}_1^\pm$ and $\tilde{\tau}_1^\pm$ masses. This last aspect is currently under study. The resolution obtained is comparable to that obtained with production threshold

studies [4,9]. A study of the mass determination errors *versus* luminosity indicates a possible saturation of the errors from the $1/\sqrt{N}$ law, occuring when the luminosity is of the order of 500 - 1000 fb^{-1} .

We would like to thank our colleagues that participated in the SUSY Snowmass-2001 studies, specially Jonathan Feng, Paul Grannis, and Robert Kahn, for their comments on this work. This work was carried out with the support of the DOE under grant DE-FG03-95ER40894.

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